

## Yukawa Induced Radiative Corrections to the Lightest Higgs Boson Mass in the NMSSM

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### Abstract

We compute the leading logarithmic radiative corrections to the lightest Higgs mass in the NMSSM involving the electroweak gauge couplings and the NMSSM specific Yukawa couplings  $\lambda$  and  $\kappa$  (including all mixed combinations), which are induced by chargino, neutralino and Higgs boson loops. The effect of the NMSSM specific Yukawa couplings  $\lambda$  and  $\kappa$  is to increase the upper bound on the lightest Higgs mass by up to  $\sim 1$  GeV, but they can also decrease the lightest Higgs mass by up to  $\sim -20$  GeV.

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# 1 Introduction

The perhaps most easily falsifiable prediction of supersymmetric extensions of the Standard Model is the prediction of a relatively light Higgs boson. In the MSSM, at tree level, the mass of the lightest CP even neutral Higgs boson is bounded from above by  $M_Z$ , which would already be ruled out by negative Higgs search results at LEP.

However, radiative corrections – mainly due to loops of quarks and squarks of the third generation – lift this upper bound considerably. In the last years, these radiative corrections to the mass of the lightest Higgs boson in the MSSM have been computed to a fairly high precision (see refs. [1–3] for recent reviews).

The knowledge of the radiative corrections to the mass of the lightest Higgs boson is important for the following reasons: Given either a lower experimental bound on its mass, or a future measurement of its mass, they allow either to decide whether a particular supersymmetric extension of the Standard Model is ruled out, or to put constraints on additional parameters of the model (as the squark masses or  $\tan\beta$ ) on which the radiative corrections depend. (This resembles to the present indirect determination of the preferred Higgs mass through precision tests in the standard model.)

In the NMSSM, the upper bound on the mass of the lightest Higgs boson is alleviated already at tree level. Subsequently we confine ourselves to the NMSSM with a scale invariant superpotential [4]

$$W = \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{\kappa}{3} \hat{S}^3 + \dots, \quad (1.1)$$

which is the only supersymmetric extension of the Standard Model where the weak scale originates from the soft susy breaking scale only, i.e. where no supersymmetric dimensionful parameters as  $\mu$  are present in the superpotential. Now, the upper bound on the mass  $M_h$  of the lightest CP even Higgs is

$$M_h^2 \leq M_Z^2 \left( \cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right) \quad (1.2)$$

which depends on the new Yukawa coupling  $\lambda$  (unless  $\sin^2 2\beta$  is very small, i.e.  $\tan\beta$  large). However, if one requires the absence of a Landau singularity for  $\lambda$  below the GUT scale,  $\lambda$  is bounded by  $\sim 0.7$  from above [4], leading again to a tree level upper bound on the mass of the lightest Higgs boson that is, however, somewhat larger than in the MSSM. Hence, as in the MSSM, the knowledge of the radiative corrections to its mass are important.

At present, these radiative corrections in the NMSSM have not been computed to the same accuracy as in the MSSM. Of course, radiative corrections in the NMSSM that are proportional to the quark/lepton Yukawa couplings and the gauge couplings only are the same as in the MSSM, but there are many additional contributions involving the new Yukawa couplings  $\lambda$  and  $\kappa$  in the superpotential in eq. (1.1), and the associated soft trilinear couplings  $A_\lambda$  and  $A_\kappa$ .

The one loop corrections in the NMSSM induced by t and b quark/squark loops have been computed already some time ago [5], and the dominant two loop corrections ( $\sim h_t^6$  and  $\sim h_t^4 \alpha_s$ ), that are the same as in the MSSM, have been included in an analysis of the NMSSM Higgs sector in ref. [6].

The leading logarithmic one loop corrections to the lightest Higgs mass in the NMSSM proportional to the electroweak gauge couplings  $g$  ( $\sim g^4$ , that are the same as in the MSSM [7]) are included in the code NMHDECAY [8], where the NMSSM Higgs masses, couplings and branching ratios are computed as functions of the parameters in the Lagrangian of the model. Recall that

large logarithmic one loop corrections  $\sim g^4$  appear as soon as the mass of a sparticle or a Higgs field is (much) larger than  $M_Z$ , and they affect the mass of the lightest Higgs boson by  $\sim -3$  GeV [7].

In this case, however, there appear also large logarithmic one loop corrections in the NMSSM  $\sim \lambda^4, g^2\lambda^2, \kappa^4$  etc, that are of the same order as the pure electroweak corrections. It is the purpose of this paper to compute these radiative corrections in the NMSSM defined by the superpotential above.

The particles whose loops generate these contributions to the lightest Higgs mass are the neutralinos, charginos and the CP even, CP odd and charged Higgs fields. There are many fields and couplings in the NMSSM that are not present in the MSSM, and which lead to the NMSSM specific radiative corrections. Their contributions to the upper bound on the lightest Higgs boson mass remain limited, however, to  $\sim 1$  GeV, if one requires the absence of a Landau singularity below  $M_{\text{GUT}}$  for all Yukawa couplings  $\lambda$ ,  $\kappa$  and  $h_t$ . On the other hand CP odd loop contributions to the effective potential can decrease the lightest Higgs boson mass by up to  $-20$  GeV.

In contrast to ref. [7] we do not use a RG analysis in an effective two Higgs doublet model below a scale  $M_A$  (as possible in the MSSM), since there are more possible mass scales in the NMSSM and the corresponding effective low energy theory is considerably more involved. Instead, we compute explicitly the contributions to the Coleman-Weinberg effective potential, the Higgs self energies and the modified running of the electroweak gauge couplings induced by heavy sparticles and/or Higgs fields. Wherever a comparison with the results of ref. [7] on the mass of the lightest CP even Higgs boson is possible, our results agree.

In the next section we compute the contributions of neutralino and chargino loops to all elements of the CP even Higgs boson mass matrix (a  $3 \times 3$  matrix in the NMSSM). The corresponding contributions from Higgs loops are, however, quite lengthy (and numerically not very important). Hence, here we just give the contributions to the mass of the lightest Higgs doublet field; these contributions can easily be translated back to contributions to the  $2 \times 2$  Higgs doublet sector of the CP even Higgs boson mass matrix. For completeness we add the contributions from the squark/slepton sectors to the modified running of the electroweak gauge couplings, that contribute also to the mass of the light Higgs doublet to  $\mathcal{O}(g^4)$ , but which are the same as in the MSSM [7].

In section 3 we study the numerical impact of the new radiative corrections on the mass of the lightest CP even Higgs boson by studying the dependence of the individual fermionic and bosonic contributions as functions of  $\tan\beta$ , and the total contributions as functions of  $\lambda$  for some specific choices of the other parameters.

## 2 Chargino/Neutralino/Higgs Loop Corrections in the NMSSM

As stated in the introduction, we consider the NMSSM with a scale invariant superpotential, which reads (for third generation quarks only, and using the conventions of ref. [8]):

$$W = \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3 + h_t \hat{Q} \hat{H}_u \hat{T}_R^c - h_b \hat{Q} \hat{H}_d \hat{B}_R^c. \quad (2.1)$$

Hereafter, hatted capital letters denote superfields, and unhatted capital letters the corresponding (complex) scalar components. The corresponding soft terms are

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = & m_{\hat{H}_u}^2 |H_u|^2 + m_{\hat{H}_d}^2 |H_d|^2 + m_S^2 |S|^2 + m_Q^2 |Q|^2 + m_T^2 |T_R|^2 + m_B^2 |B_R|^2 \\ & + (\lambda A_\lambda H_u H_d S + \frac{1}{3} \kappa A_\kappa S^3 + h_t A_t Q H_u T_R^c - h_b A_b Q H_d B_R^c + \text{h.c.}) . \end{aligned} \quad (2.2)$$

Subsequently we denote the neutral CP even Higgs vevs by  $h_u$ ,  $h_d$  and  $s$ . It is also convenient to introduce the quantities  $\mu = \lambda s$  and  $\nu = \kappa s$ .

First we consider the loop corrections induced by charginos and neutralinos. These contribute to three distinct effects that have to be added up:

i) First, they contribute to the Coleman-Weinberg effective potential (in the  $\overline{\text{DR}}$  scheme which is, however, irrelevant for the leading logarithms)

$$\Delta V_{\text{eff}} = \frac{1}{64\pi^2} \text{STr} M^4 \left[ \ln \left( \frac{M^2}{Q^2} \right) - \frac{3}{2} \right]. \quad (2.3)$$

Subsequently we will choose  $Q^2 = M_Z^2$ , hence all couplings and parameters in eqs. (2.1) and (2.2) are renormalized at the scale  $M_Z^2$ .

ii) Second, they contribute to the Higgs self energies, which we can compute at vanishing momenta within the present leading logarithmic approximation.

iii) Third, they affect the running of the electroweak gauge couplings at scales below their (susy breaking) masses: Only at scales above all susy breaking masses we can identify the electroweak gauge couplings with the parameters  $\hat{g}_1$ ,  $\hat{g}_2$  that appear as D-terms in the tree level Higgs potential in the form  $(h_u^2 - h_d^2)^2(\hat{g}_1^2 + \hat{g}_2^2)/8$ . Quantum effects contributing to the Higgs potential (in terms of properly normalized fields) are taken care of through the contributions i) and ii) above, but at the end we want to express the Higgs masses in terms of electroweak gauge couplings at the scale  $M_Z$  (and not at  $M_{\text{susy}}$ ). This latter effect has to be treated separately.

The mass matrices  $M_{\chi^\pm}$  and  $M_{\chi^0}$  for the charginos and neutralinos are (using the conventions of ref. [8])

$$M_{\chi^\pm} = \begin{pmatrix} M_2 & g_2 h_u \\ g_2 h_d & \mu \end{pmatrix} \quad (2.4)$$

and

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & \frac{g_1 h_u}{\sqrt{2}} & -\frac{g_1 h_d}{\sqrt{2}} & 0 \\ & M_2 & -\frac{g_2 h_u}{\sqrt{2}} & \frac{g_2 h_d}{\sqrt{2}} & 0 \\ & & 0 & -\mu & -\lambda h_d \\ & & & 0 & -\lambda h_u \\ & & & & 2\nu \end{pmatrix} \quad (2.5)$$

respectively. Here  $M_1$  and  $M_2$  are soft susy breaking  $U(1)$  and  $SU(2)$  gaugino masses.

Large logarithms appear as soon as any of the charginos and/or neutralinos has a mass much larger than  $M_Z$ . Accordingly we can expect the presence of the following potentially large logarithms (here we do not distinguish between  $M_1$  and  $M_2$ , whose ratio we do not consider as exponentially large):

$$\begin{aligned} L_M &= \ln(M_2^2/M_Z^2) \sim \ln(M_1^2/M_Z^2) \\ L_\mu &= \ln(\mu^2/M_Z^2) \\ L_\nu &= \ln(4\nu^2/M_Z^2) \\ L_1 &= \ln(\text{Max}(M_2^2, \mu^2)/M_Z^2) \\ L_2 &= \ln(\text{Max}(\mu^2, 4\nu^2)/M_Z^2). \end{aligned} \quad (2.6)$$

Next we give the sum of the three contributions i), ii) and iii) above to the 6 matrix elements  $M_{uu}^2$ ,  $M_{dd}^2$ ,  $M_{ud}^2$ ,  $M_{ss}^2$ ,  $M_{us}^2$ ,  $M_{ds}^2$  of the neutral CP even Higgs mass matrix in the NMSSM (in

an obvious notation). To this end we define  $g^2 = \frac{g_1^2 + g_2^2}{2}$ ,  $v^2 = h_u^2 + h_d^2$  (hence  $M_Z^2 = g^2 v^2$ ) and  $\tan\beta = h_u/h_d$ . Then we obtain

$$\begin{aligned}\Delta M_{uu}^2 &= \frac{1}{16\pi^2} \left( -\frac{8}{3} g^4 v^2 \sin^2 \beta \cos^4 \theta_W L_M \right. \\ &\quad + \left[ -\frac{4}{3} g^4 v^2 \sin^2 \beta (2 \sin^4 \theta_W - 2 \sin^2 \theta_W + 1) + 2\lambda\kappa\mu^2 \cot \beta \right] L_\mu \\ &\quad + 2\lambda\kappa\nu^2 \cot \beta L_\nu + \left[ g^4 v^2 \sin^2 \beta (-16 \sin^4 \theta_W + 28 \sin^2 \theta_W - 14) \right. \\ &\quad \left. + g^2 \mu\nu \cot \beta (-2 \sin^2 \theta_W + 3) \right] L_1 - 3\lambda\kappa\mu^2 \cot \beta L_2 \Big) ,\end{aligned}\tag{2.7}$$

$$\begin{aligned}\Delta M_{dd}^2 &= \frac{1}{16\pi^2} \left( -\frac{8}{3} g^4 v^2 \cos^2 \beta \cos^4 \theta_W L_M \right. \\ &\quad + \left[ -\frac{4}{3} g^4 v^2 \cos^2 \beta (2 \sin^4 \theta_W - 2 \sin^2 \theta_W + 1) + 2\lambda\kappa\mu^2 \tan \beta \right] L_\mu \\ &\quad + 2\lambda\kappa\nu^2 \tan \beta L_\nu + \left[ g^4 v^2 \cos^2 \beta (-16 \sin^4 \theta_W + 28 \sin^2 \theta_W - 14) \right. \\ &\quad \left. + g^2 \mu\nu \tan \beta (-2 \sin^2 \theta_W + 3) \right] L_1 - 3\lambda\kappa\mu^2 \tan \beta L_2 \Big) ,\end{aligned}\tag{2.8}$$

$$\begin{aligned}\Delta M_{ud}^2 &= \frac{1}{16\pi^2} \left( \frac{4}{3} g^4 v^2 \sin 2\beta \cos^4 \theta_W L_M \right. \\ &\quad + \left[ \frac{2}{3} g^4 v^2 \sin 2\beta (2 \sin^4 \theta_W - 2 \sin^2 \theta_W + 1) - 2\lambda\kappa\mu^2 \right] L_\mu \\ &\quad - 2\lambda\kappa\nu^2 L_\nu + \left[ g^4 v^2 \sin 2\beta (2 \sin^2 \theta_W - 5) \right. \\ &\quad + g^2 v^2 \lambda^2 \sin 2\beta (-2 \sin^2 \theta_W + 3) + g^2 \mu\nu (2 \sin^2 \theta_W - 3) \Big] L_1 \\ &\quad \left. + \left[ -g^2 v^2 \lambda^2 \sin 2\beta + 3\lambda\kappa\mu^2 \right] L_2 \right) ,\end{aligned}\tag{2.9}$$

$$\Delta M_{ss}^2 = \frac{1}{16\pi^2} \left( 8\mu^2 (\kappa^2 - \lambda^2) L_\mu - 16\kappa^2 \nu^2 L_\nu \right) ,\tag{2.10}$$

$$\begin{aligned}\Delta M_{us}^2 &= \frac{v\mu}{16\pi^2} \left( 4\lambda^2 (\lambda \sin \beta - \kappa \cos \beta) L_\mu + 4\kappa^2 (\lambda \sin \beta - \kappa \cos \beta) L_\nu \right. \\ &\quad + g^2 (\lambda \sin \beta + \kappa \cos \beta) (4 \sin^2 \theta_W - 6) L_1 \\ &\quad \left. + \lambda (-16\kappa^2 \sin \beta - 2\lambda^2 \sin \beta + 6\lambda\kappa \cos \beta) L_2 \right) ,\end{aligned}\tag{2.11}$$

$$\begin{aligned}\Delta M_{ds}^2 &= \frac{v\mu}{16\pi^2} \left( 4\lambda^2 (\lambda \cos \beta - \kappa \sin \beta) L_\mu + 4\kappa^2 (\lambda \cos \beta - \kappa \sin \beta) L_\nu \right. \\ &\quad + g^2 (\lambda \cos \beta + \kappa \sin \beta) (4 \sin^2 \theta_W - 6) L_1 \\ &\quad \left. + \lambda (-16\kappa^2 \cos \beta - 2\lambda^2 \cos \beta + 6\lambda\kappa \sin \beta) L_2 \right) .\end{aligned}\tag{2.12}$$

In the doublet sector of the neutral CP even Higgs sector one can identify a state whose mass is bounded from above by  $M_Z$  at tree level in the MSSM, and by eq. (1.2) in the NMSSM. The radiative corrections to this upper bound are quite important, and are given by  $\Delta M_h^2 = \sin^2 \beta \Delta M_{uu}^2 + \cos^2 \beta \Delta M_{dd}^2 + \sin 2\beta \Delta M_{ud}^2$ . From the chargino and neutralino loops in the NMSSM we obtain

$$\begin{aligned}\Delta^{\text{ferm}} M_h^2 &= \frac{v^2}{16\pi^2} \left( -\frac{8}{3} g^4 \cos^2 2\beta \cos^4 \theta_W L_M \right. \\ &\quad \left. - \frac{4}{3} g^4 \cos^2 2\beta (2 \sin^4 \theta_W - 2 \sin^2 \theta_W + 1) L_\mu \right.\end{aligned}$$

$$\begin{aligned}
& + \left[ g^4 \left\{ \sin^2 2\beta (8 \sin^4 \theta_W - 12 \sin^2 \theta_W + 2) - 16 \sin^4 \theta_W + 28 \sin^2 \theta_W - 14 \right\} \right. \\
& \left. + g^2 \lambda^2 \sin^2 2\beta (-2 \sin^2 \theta_W + 3) \right] L_1 - g^2 \lambda^2 \sin^2 2\beta L_2 \Big) . \tag{2.13}
\end{aligned}$$

This and the previous results agree, for  $\lambda \rightarrow 0$ , with the ones in ref. [7].

Now we turn to the contributions from the charged, neutral CP even and CP odd Higgs loops. These contribute to the Coleman-Weinberg effective potential and to the modification of the running of the electroweak gauge couplings, but their contributions to the Higgs self energies do not give raise to large logarithms. The corresponding tree level mass matrices  $M_{\pm}^2$ ,  $M_S^2$  and  $M_P^2$  are, respectively,

$$M_{\pm}^2 = \begin{pmatrix} m_{H_u}^2 + \mu^2 + \frac{g^2(h_u^2 - h_d^2)}{2} + \frac{g_2^2 h_d^2}{2} & h_u h_d \left( \frac{g_2^2}{2} - \lambda^2 \right) + \mu(\nu + A_\lambda) \\ h_u h_d \left( \frac{g_2^2}{2} - \lambda^2 \right) + \mu(\nu + A_\lambda) & m_{H_d}^2 + \mu^2 + \frac{g^2(h_d^2 - h_u^2)}{2} + \frac{g_2^2 h_u^2}{2} \end{pmatrix} , \tag{2.14}$$

$$\begin{aligned}
M_{S,11}^2 &= m_{H_u}^2 + \mu^2 + \lambda^2 h_d^2 + \frac{g^2}{2}(3h_u^2 - h_d^2) \\
M_{S,22}^2 &= m_{H_d}^2 + \mu^2 + \lambda^2 h_u^2 + \frac{g^2}{2}(3h_d^2 - h_u^2) \\
M_{S,12}^2 &= h_u h_d (2\lambda^2 - g^2) - \mu(A_\lambda + \nu) \\
M_{S,33}^2 &= m_S^2 + \lambda^2(h_u^2 + h_d^2) + 6\nu^2 - 2\lambda\kappa h_u h_d + 2\nu A_\kappa \\
M_{S,13}^2 &= 2\lambda\mu h_u - \lambda h_d(2\nu + A_\lambda) \\
M_{S,23}^2 &= 2\lambda\mu h_d - \lambda h_u(2\nu + A_\lambda) , \tag{2.15}
\end{aligned}$$

$$\begin{aligned}
M_{P,11}^2 &= m_{H_u}^2 + \mu^2 + \frac{g^2}{2}(h_u^2 - h_d^2) \\
M_{P,22}^2 &= m_{H_d}^2 + \mu^2 + \frac{g^2}{2}(h_d^2 - h_u^2) \\
M_{P,12}^2 &= \mu(A_\lambda + \nu) \\
M_{P,33}^2 &= m_S^2 + \lambda^2(h_u^2 + h_d^2) + 2\nu^2 + 2\lambda\kappa h_u h_d - 2\nu A_\kappa \\
M_{P,13}^2 &= \lambda h_d(A_\lambda - 2\nu) \\
M_{P,23}^2 &= \lambda h_u(A_\lambda - 2\nu) . \tag{2.16}
\end{aligned}$$

These mass matrices have to be used in the Coleman-Weinberg effective potential eq. (2.3) and in the computation of its derivatives w.r.t.  $h_u$ ,  $h_d$  and  $s$  that contribute to the neutral CP even mass matrix; only then they can be simplified by applying the minimization equations of the tree level potential

$$\begin{aligned}
m_{H_u}^2 &= -\mu^2 - \lambda^2 h_d^2 + \frac{g^2}{2}(h_d^2 - h_u^2) + \cot \beta \mu(A_\lambda + \nu) , \\
m_{H_d}^2 &= -\mu^2 - \lambda^2 h_u^2 + \frac{g^2}{2}(h_u^2 - h_d^2) + \tan \beta \mu(A_\lambda + \nu) , \\
m_S^2 &= -\lambda^2(h_u^2 + h_d^2) - 2\nu^2 + 2\lambda\kappa h_u h_d - \nu A_\kappa + \lambda A_\lambda \frac{h_u h_d}{s} \tag{2.17}
\end{aligned}$$

in order to eliminate the soft masses squared.

As before, large logarithms appear in the loop contributions to the neutral CP even mass matrix, if any of the eigenstates of the three mass matrices above is much larger than  $M_Z$ .

In the charged Higgs sector, the diagonalization of the  $2 \times 2$  mass matrix is straightforward. As in the MSSM, an eigenstate (apart from the Goldstone boson) is much heavier than  $M_Z$  if (using eq. (2.17))

$$M_A^2 \equiv \mu(A_\lambda + \nu) \frac{(h_u^2 + h_d^2)}{h_u h_d} \quad (2.18)$$

satisfies

$$M_A \gg M_Z . \quad (2.19)$$

(Here we introduced the MSSM-like notation  $M_A$  for the mass of the heavy Higgs doublet in the limit (2.19).) Accordingly a potentially large logarithm is given by  $L_A = \ln(M_A^2/M_Z^2)$ .

In the neutral CP even sector, the situation is considerably more involved, since we have to deal with a  $3 \times 3$  mass matrix. It is convenient to perform first a preliminary approximate diagonalization by defining  $\widetilde{M}_S^2 = \mathcal{O}^T M_S^2 \mathcal{O}$  with

$$\mathcal{O} = \begin{pmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} . \quad (2.20)$$

(After the computation of the derivatives of the effective potential w.r.t. the vevs, the angle  $\beta$  can be identified with  $\arctan(h_u/h_d)$  as in the MSSM.)

In the limit (2.19) we have  $\widetilde{M}_{S,11}^2 \simeq M_A^2$ , whereas  $\widetilde{M}_{S,22}^2$  is bounded from above by eq. (1.2). The element  $\widetilde{M}_{S,33}^2 \equiv M_{SS}^2$  corresponds to the mass squared of the CP even singlet state in the decoupling limit  $\lambda \rightarrow 0$ , that can also be much larger than  $M_Z$  if (using eq. (2.17))

$$M_{SS}^2 \simeq \nu(4\nu + A_\kappa) \gg M_Z^2 . \quad (2.21)$$

This leads to the appearance of another potentially large logarithm  $L_S = \ln(M_{SS}^2/M_Z^2)$ .

Now the CP even mass matrix  $\widetilde{M}_S^2$  can be diagonalized perturbatively in powers of  $M_Z/M_A$  and  $M_Z/M_{SS}$ . (Recall that these ratios have to be small whenever large logarithms appear.) During this perturbative diagonalization we have to keep all terms of  $\mathcal{O}(M_Z^4)$  or larger in the expression  $\text{Tr} M_S^4$ , that appears as coefficient of potentially large logarithms in the effective potential (2.3).

A priori this leads to quite lengthy expressions since, for  $M_A \gg M_Z$ , the off-diagonal elements  $\widetilde{M}_{S,13}^2$  and  $\widetilde{M}_{S,23}^2$  are of  $\mathcal{O}(M_A M_Z)$ . The absence of a negative eigenvalue of  $\widetilde{M}_S^2$  (that would signal an instability of the tree level potential) leads, however, to some simplifications: The subdeterminant in the (2,3)-sector, given by  $\widetilde{M}_{S,22}^2 \widetilde{M}_{S,33}^2 - \widetilde{M}_{S,23}^4$ , must be positive. Iff  $\widetilde{M}_{S,23}^2 \sim \mathcal{O}(M_A M_Z) \gg M_Z^2$ , it follows from  $\widetilde{M}_{S,22}^2 \sim \mathcal{O}(M_Z^2)$  that  $\widetilde{M}_{S,33}^2 \gtrsim M_A^2$  and hence  $M_Z/M_{SS}$  is at least as small as  $M_Z/M_A$ . On the other hand, recalling the definitions of  $\mu = \lambda s$  and  $\nu = \kappa s$ , one finds that  $M_{SS} \gg M_A$  is possible only for  $\lambda \ll \kappa$ , in which case the singlet sector decouples and the off-diagonal elements  $\widetilde{M}_{S,13}^2$  and  $\widetilde{M}_{S,23}^2$  can never be large. The resulting contribution from neutral CP even Higgs loops to the effective potential will be given below.

The situation in the neutral CP odd sector is similarly complex. Again, it is convenient to perform a first preliminary approximate diagonalization  $\widetilde{M}_P^2 = \widehat{\mathcal{O}}^T M_P^2 \widehat{\mathcal{O}}$  where  $\widehat{\mathcal{O}}$  differs from  $\mathcal{O}$  in eq. (2.20) by  $\sin \beta \rightarrow -\sin \beta$ .

Let us now review the potentially large matrix elements (with respect to  $M_Z^2$ ). The element  $\widetilde{M}_{P,11}^2$  is of  $\mathcal{O}(M_A^2)$  in the limit (2.19). The element  $\widetilde{M}_{P,33}^2$  corresponds to the mass  $M_{PS}$  squared of the CP odd singlet state in the decoupling limit  $\lambda \rightarrow 0$ .  $M_{PS}$  can be much larger than  $M_Z$  if

$$M_{PS}^2 \simeq -3\nu A_\kappa \gg M_Z^2 \quad (2.22)$$

(hence  $-3\nu A_\kappa$  must be positive in this limit). Finally the element  $\widetilde{M}_{P,13}^2$  can also be large. Next we proceed in two steps:

First, we semi-diagonalize the  $3 \times 3$  matrix  $\widetilde{M}_P^2$  into the above potentially large  $2 \times 2$  submatrix and the Goldstone sector, perturbatively in powers of  $M_Z$  as in the CP even case. The result is not quite the same as in the CP even sector, however: In the CP odd case the matrix element  $\widetilde{M}_{P,23}^2$  is never  $\gg M_Z^2$ . This leads to the absence of certain terms  $\sim \widetilde{M}_{P,23}^2$  as compared to the CP even case.

Second, we diagonalize exactly the potentially large  $2 \times 2$  submatrix consisting of  $\widetilde{M}_{P,11}^2$ ,  $\widetilde{M}_{P,33}^2$  and  $\widetilde{M}_{P,13}^2$ . The corresponding eigenvalues will be denoted by  $E_+^2$  and  $E_-^2$ . (Note that, for  $M_{PS}^2 \lesssim M_A^2$ , we have  $E_+^2 \sim M_A^2$ .) Potentially large logarithms are now  $\ln(E_+^2/M_Z^2)$  and  $\ln(E_+^2/E_-^2)$ . The second logarithm is large only for  $E_+^2 \gg E_-^2$ , in which case we have  $E_+^2 \sim \widetilde{M}_{P,11}^2 + \widetilde{M}_{P,33}^2$ ,  $E_-^2 \sim (\widetilde{M}_{P,11}^2 \widetilde{M}_{P,33}^2 - \widetilde{M}_{P,13}^4)/E_+^2$  and the rotation angle  $\gamma$  satisfies  $\sin^2 \gamma \sim \widetilde{M}_{P,13}^2/E_+^2$ .

We will now present, as an intermediate result, the leading logarithmic contributions to the effective potential (2.3) from Higgs boson loops in terms of the Higgs boson mass matrix elements. (Here  $\widetilde{M}_\pm^2$  denotes the charged Higgs mass matrix after the rotation by  $\mathcal{O}_{2 \times 2}$ , with  $\mathcal{O}_{2 \times 2}$  given by the upper left part of  $\mathcal{O}$  in eq. (2.20).) The list of all potentially large logarithms is given by

$$\begin{aligned} L_A &= \ln(M_A^2/M_Z^2) \\ L_S &= \ln(M_{SS}^2/M_Z^2) \\ L_{MAS} &= \ln(\max(M_A^2, M_{SS}^2)/M_Z^2) \\ L_{E_+} &= \ln(E_+^2/M_Z^2) \\ L_{E_+E_-} &= \ln(E_+^2/E_-^2) . \end{aligned} \quad (2.23)$$

Then we obtain

$$\begin{aligned} \Delta V_{\text{eff}}^{\text{bos}} &= \frac{1}{64\pi^2} \left( \left[ 2\widetilde{M}_{\pm,11}^4 + 4\widetilde{M}_{\pm,12}^4 + \widetilde{M}_{S,11}^4 + 2\widetilde{M}_{S,12}^4 \right] L_A \right. \\ &\quad + \left[ \widetilde{M}_{S,33}^4 + 2\widetilde{M}_{S,23}^4 + 2\widetilde{M}_{S,23}^2 \widetilde{M}_{S,22}^2/M_{SS}^2 - \widetilde{M}_{S,23}^8/M_{SS}^4 \right] L_S \\ &\quad + \left[ \widetilde{M}_{P,11}^4 + \widetilde{M}_{P,33}^4 + 2\widetilde{M}_{P,12}^4 + 2\widetilde{M}_{P,13}^4 + 2\widetilde{M}_{P,23}^4 \right] L_{E_+} \\ &\quad \left. - \left[ E_-^4 + 2(\sin \gamma \widetilde{M}_{P,12}^2 - \cos \gamma \widetilde{M}_{P,23}^2)^2 \right] L_{E_+E_-} + 2\widetilde{M}_{S,13}^4 L_{MAS} \right) . \end{aligned} \quad (2.24)$$

(The terms with  $M_{SS}$  in the denominator can still be of  $\mathcal{O}(M_Z^4)$ , given the corresponding sizes of the off-diagonal matrix elements. These terms originate from the perturbative diagonalization of the CP even mass matrix.)

Finally, the effect of a heavy Higgs doublet on the modification of the running of the electroweak gauge couplings below  $M_A$  can be written in terms of a contribution to the effective potential that is the same as in the MSSM [7]:

$$\Delta V_{\text{run}}^{\text{bos}} = \frac{g^4}{192\pi^2} (h_u^2 - h_d^2)^2 (-1 + 2\sin^2 \theta_W - 2\sin^4 \theta_W) L_A . \quad (2.25)$$



It is straightforward, in principle, to compute the derivatives of  $\Delta V_{\text{eff}}^{\text{bos}}$  and  $\Delta V_{\text{run}}^{\text{bos}}$  w.r.t.  $h_u, h_d$  and  $s$ , and hence the contributions to the neutral CP even Higgs mass matrix, given the explicit expressions (2.14), (2.15) and (2.16) for the Higgs mass matrix elements. The explicit form of these contributions to all six matrix elements are very lengthy, however. Subsequently we confine ourselves to the contributions to the mass squared of the lightest neutral CP even Higgs boson obtained from  $\Delta V_{\text{eff}}^{\text{bos}}$  and  $\Delta V_{\text{run}}^{\text{bos}}$ . These simplify considerably, and can be written as

$$\begin{aligned}
\Delta^{\text{bos}} M_h^2 = & \frac{v^2}{16\pi^2} \left( \left[ \frac{g^4}{3} (\sin^4 \theta_W (4 + 2 \sin^2 2\beta) - \sin^2 \theta_W (4 + 8 \sin^2 \beta)) \right. \right. \\
& - \frac{33}{4} \sin^4 2\beta + 16 \sin^2 2\beta + \frac{5}{4} \Big) + g^2 \lambda^2 \left( 2 \sin^2 \theta_W \sin^2 2\beta + \frac{11}{2} \sin^4 2\beta - \frac{15}{2} \sin^2 2\beta - 1 \right) \\
& + \lambda^4 \left( -\frac{11}{4} \sin^4 2\beta + \frac{5}{4} \sin^2 2\beta + 1 \right) \Big] L_A \\
& + \left[ \lambda^2 (\lambda - \kappa \sin 2\beta)^2 + \frac{3\lambda^2}{M_{SS}^2} (g^2 \cos^2 2\beta + \lambda^2 \sin^2 2\beta) (2\mu - \sin 2\beta (A_\lambda + 2\nu))^2 \right. \\
& - \frac{\lambda^4}{M_{SS}^4} (2\mu - \sin 2\beta (A_\lambda + 2\nu))^4 \Big] L_S \\
& + \left[ \frac{g^4}{4} (1 - \sin^4 2\beta) + g^2 \lambda^2 \left( \frac{1}{2} \sin^4 2\beta + \frac{1}{2} \sin^2 2\beta - 1 \right) \right. \\
& + \lambda^4 \left( -\frac{1}{4} \sin^4 2\beta - \frac{1}{2} \sin^2 2\beta + 1 \right) + \lambda^2 (\lambda + \kappa \sin 2\beta)^2 \Big] L_{E_+} \\
& + \left[ \frac{\lambda^2 - g^2}{2} \sin^2 2\beta \cos^2 2\beta \frac{M_{PS}^2}{E_+^2} \right. \\
& - \left( M_A^2 \lambda (\lambda + \kappa \sin 2\beta) - \lambda^2 (A_\lambda - 2\nu)^2 - \frac{M_{PS}^2}{2} (g^2 \cos^2 2\beta - \lambda^2 (1 + \cos^2 2\beta)) \right)^2 / E_+^4 \\
& \left. + M_A^2 M_{PS}^2 \lambda (\lambda + \kappa \sin 2\beta) (g^2 \cos^2 2\beta - \lambda^2 (1 + \cos^2 2\beta)) / E_+^4 \right] L_{E_+ E_-} \Big) . \tag{2.26}
\end{aligned}$$

$M_A$ ,  $M_{SS}$  and  $M_{PS}$  are given in eqs. (2.18), (2.21) and (2.22), respectively. We recall that the terms with  $M_{SS}$  or  $E_+$  in the denominators are relevant in the limit  $M_{SS}, E_+ \gg M_Z$  only and, in any case, are valid only if all eigenvalues of the CP even and odd tree level mass squared matrices are positive. For  $\lambda \rightarrow 0$  the result of eq. (2.26) coincides again with the one in ref. [7] obtained in the MSSM.

The contribution (2.26) to  $\Delta M_h^2$  can be translated back to contributions to the CP even neutral Higgs mass matrix elements according to the rules

$$\begin{aligned}
\Delta^{\text{bos}} M_{uu}^2 &= \sin^2 \beta \Delta^{\text{bos}} M_h^2 \\
\Delta^{\text{bos}} M_{dd}^2 &= \cos^2 \beta \Delta^{\text{bos}} M_h^2 \\
\Delta^{\text{bos}} M_{ud}^2 &= \sin \beta \cos \beta \Delta^{\text{bos}} M_h^2
\end{aligned} \tag{2.27}$$

which gives the correct results in the limit where both  $M_A$  and  $M_{SS}$  are much larger than  $M_Z$ , and the corrections to the masses of the heavy Higgs states are negligible. (The resulting expressions for  $\Delta^{\text{bos}} M_{uu}^2$ ,  $\Delta^{\text{bos}} M_{dd}^2$ ,  $\Delta^{\text{bos}} M_{ud}^2$  differ from the ones in ref. [7], i.e. our method gives different corrections to the mass of the heavy Higgs states.)

For completeness we give also the contribution of sfermions (squarks and sleptons)  $\sim g^4$  to the mass of the lightest CP even neutral Higgs boson, which are again the same as in the MSSM. As in ref. [7] we assume a common mass  $M_{\text{susy}}$  for all squarks and sleptons, with the exception of the squarks of the third generation whose masses are denoted by  $m_Q$ ,  $m_T$  and  $m_B$ . Defining

$$\begin{aligned} \Delta^{\text{sferm}} = & \frac{v^2 g^4}{12\pi^2} \left\{ \left( \frac{3}{2} - 3\sin^2\theta_W + \frac{5}{3}\sin^4\theta_W \right) \ln(m_Q^2/M_Z^2) \right. \\ & + \frac{4}{3}\sin^4\theta_W \ln(m_T^2/M_Z^2) + \frac{1}{3}\sin^4\theta_W \ln(m_B^2/M_Z^2) \\ & \left. + \left( \frac{9}{2} - 9\sin^2\theta_W + \frac{38}{3}\sin^4\theta_W \right) \ln(M_{\text{susy}}^2/M_Z^2) \right\} \end{aligned} \quad (2.28)$$

one finds

$$\begin{aligned} \Delta^{\text{sferm}} M_{uu}^2 &= \sin^2\beta \Delta^{\text{sferm}} \\ \Delta^{\text{sferm}} M_{dd}^2 &= \cos^2\beta \Delta^{\text{sferm}} \\ \Delta^{\text{sferm}} M_{ud}^2 &= -\sin\beta \cos\beta \Delta^{\text{sferm}} \end{aligned} \quad (2.29)$$

in agreement with ref. [7].

### 3 Impact on the Mass of the Lightest Neutral Higgs Boson

In the present paper we have computed all leading logarithmic radiative corrections to the lightest Higgs mass in the NMSSM, to fourth order in the electroweak gauge couplings,  $\lambda$  and  $\kappa$  (including all mixed combinations). The essential results are the radiative corrections (2.13) and (2.26) to the mass squared of the lightest CP even neutral Higgs boson induced by chargino, neutralino and Higgs boson loops. The numerical impact on its mass  $\Delta M_h$  can most easily be studied using

$$\Delta M_h \simeq \frac{1}{2M_h} \Delta M_h^2. \quad (3.1)$$

Subsequently we want to plot the numerical effect of the radiative corrections considered here as a function of  $\tan\beta$  or  $\lambda$ . However, since  $M_h$  depends on  $\tan\beta$  and  $\lambda$  already at tree level, the straightforward use of eq. (3.1) to compute  $\Delta M_h$  would not allow to disentangle the effect due to the radiative corrections from the effect due to the tree level variation of  $M_h$ . In addition, radiative corrections from the top/stop sector on  $M_h$  in the denominator of eq. (3.1), which we do not consider in this paper, are also very important in the NMSSM. Therefore, for the purpose to focus on the effect due to the radiative corrections in eq. (3.1), we fix the numerical value of  $M_h$  in the denominator to  $M_h = 120$  GeV, just above the lower limit from LEP. The numerical values for  $\Delta M_h$  obtained below can easily be rescaled to any other value of  $M_h$  using the relation above. (Notably they will decrease for larger values of  $M_h$ .)

First we are interested in maximizing the upper bound on the lightest Higgs mass. Whereas the fermionic contributions (2.13) to  $M_h^2$  are generally negative (and small for chargino and neutralino masses  $\sim M_Z$ ), the bosonic contribution (2.26) can be marginally positive. The following choice of parameters (for soft terms bounded by 1 TeV, and  $\lambda$  and  $\kappa$  bounded by the absence of a Landau pole below  $M_{\text{GUT}}$ ) renders the sum of the fermionic and bosonic contributions as positive as possible:  $\lambda = 0.1$ ,  $\kappa = 0.5$ ,  $\mu = M_2 = 100$  GeV,  $A_\lambda = 1$  TeV and  $A_\kappa = -100$  GeV. In Fig. 1 we plot  $\Delta^{\text{ferm}} M_h$ ,

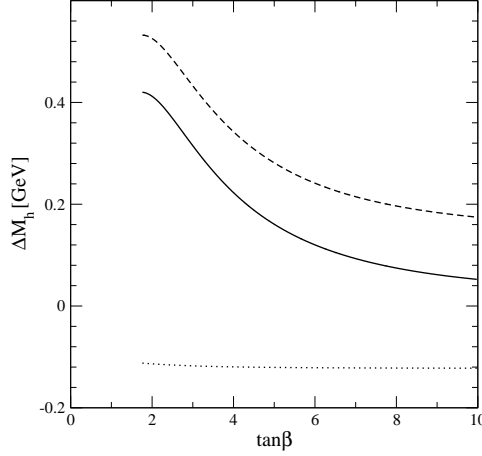


Figure 1:  $\Delta^{\text{ferm}} M_h$  (dotted line),  $\Delta^{\text{bos}} M_h$  (dashed line) and the sum (full line) as a function of  $\tan\beta$  for  $\lambda = 0.1$ ,  $\kappa = 0.5$ ,  $\mu = M_2 = 100$  GeV,  $A_\lambda = 1$  TeV and  $A_\kappa = -100$  GeV.

$\Delta^{\text{bos}} M_h$  and the sum as a function of  $\tan\beta$ . Values of  $\tan\beta \lesssim 1.5$  are omitted since they would give raise to a Landau Pole for the top quark Yukawa coupling. We see that the maximal increase in  $M_h$  is just  $\sim 0.5$  GeV.

In order to see the impact of the NMSSM specific contributions  $\sim \lambda$  and  $\sim \kappa$ , we plot in Fig. 2  $\Delta M_h$  as a function of  $\lambda$  (keeping  $\kappa = \lambda$ ) for  $\tan\beta = 1.5$ . The other parameters are such that the positive NMSSM specific effect is maximized:  $\mu = 500$  GeV,  $M_2 = 1$  TeV,  $A_\lambda = -250$  GeV, and  $A_\kappa = -1$  TeV. Values of  $\lambda$  larger than 0.5 would correspond to a Landau Pole below the GUT scale. Here one sees that the NMSSM specific contributions to  $M_h$  (the difference between  $\lambda = \kappa = 0.5$  and  $\lambda = \kappa = 0$ ) are marginally positive, up to +1 GeV within the allowed range for the Yukawa couplings.

However, the NMSSM specific contributions to the lightest Higgs mass can also be negative, and some of them can be quite large in absolute value. These contributions originate from the CP odd sector and are of the form  $-(v^2/16\pi^2)\lambda^4(A_\lambda - 2\nu)^4/E_+^4 L_{E_+E_-}$  (obtained by expanding the square of the next-to-last line in eq. (2.26)). If  $|A_\lambda - 2\nu| \gg E_+$  (keeping  $E_+ > M_Z$  and a positive determinant for the CP odd mass matrix), this contribution can lower the lightest Higgs mass by up to  $\sim -20$  GeV. The corresponding phenomenon is shown in Fig. 3, which gives the total contribution to  $\Delta M_h$  as a function of  $\lambda = \kappa$  for  $\tan\beta = 2$ ,  $\mu = M_2 = 1$  TeV,  $A_\lambda = -800$  GeV, and  $A_\kappa = -50$  GeV.

To conclude, the radiative corrections to the lightest Higgs mass in the NMSSM considered here are typically negative (as large as  $-20$  GeV, if  $A_\lambda$  and  $\nu$  differ in sign), but can increase its upper bound only by up to +1 GeV.

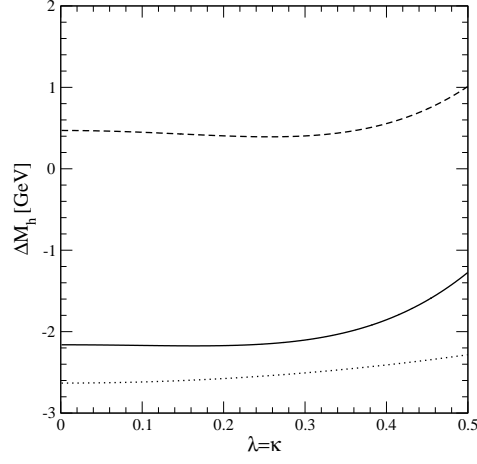


Figure 2:  $\Delta^{\text{ferm}} M_h$  (dotted line),  $\Delta^{\text{bos}} M_h$  (dashed line) and the sum (full line) as a function of  $\lambda = \kappa$  for  $\tan\beta = 1.5$ ,  $\mu = 500$  GeV,  $M_2 = 1$  TeV,  $A_\lambda = -250$  GeV, and  $A_\kappa = -1$  TeV

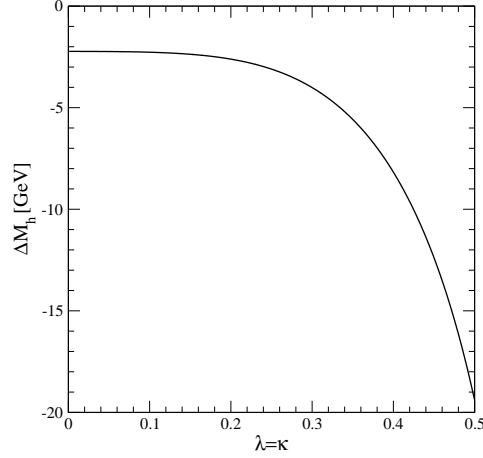


Figure 3: Total contribution to  $\Delta M_h$  as a function of  $\lambda = \kappa$  for  $\tan\beta = 2$ ,  $\mu = M_2 = 1$  TeV,  $A_\lambda = -800$  GeV, and  $A_\kappa = -50$  GeV

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